

# Descendants of algebra 5.12 of order $p^7$

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We have two four parameter families of descendants of algebra 5.12 of order  $p^7$ . The parameters are  $x, y, z, t$  in both cases.

## 1 Note 1

We put the parameters  $x, y, z, t$  in a matrix  $\begin{pmatrix} x & y \\ z & t \end{pmatrix}$ , and the distinct algebras correspond to orbits of matrices  $A = \begin{pmatrix} x & y \\ z & t \end{pmatrix}$  with entries in  $\text{GF}(p)$  under the action

$$A \rightarrow \frac{1}{\det P} P A P^{-1}$$

where  $P$  is the subgroup of  $\text{GL}(2, p)$  consisting of non-singular matrices  $\begin{pmatrix} \alpha & \beta \\ \beta & \alpha \end{pmatrix}$  or  $\begin{pmatrix} \alpha & \beta \\ -\beta & -\alpha \end{pmatrix}$ . So we want to pick out a set of orbit representatives. Notes5.12.m is a MAGMA program which outputs a matrix mats1 with suitable  $[x, y, z, t]$  as rows.

## 2 Note 2

We put the parameters  $x, y, z, t$  in a matrix  $\begin{pmatrix} x & y \\ z & t \end{pmatrix}$ , and the distinct algebras correspond to orbits of matrices  $A = \begin{pmatrix} x & y \\ z & t \end{pmatrix}$  with entries in  $\text{GF}(p)$  under the action

$$A \rightarrow \frac{1}{\det P} P A P^{-1}$$

where  $P$  is the subgroup of  $\text{GL}(2, p)$  consisting of non-singular matrices  $\begin{pmatrix} \alpha & \omega\beta \\ \beta & \alpha \end{pmatrix}$  or  $\begin{pmatrix} \alpha & \omega\beta \\ -\beta & -\alpha \end{pmatrix}$ . So we want to pick out a set of orbit representatives. Notes5.12.m is a MAGMA program which outputs a matrix mats2 with suitable  $[x, y, z, t]$  as rows.