

Algebra 6.114

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Algebra 6.114 has presentation

$$\langle a, b, c \mid pa - ba, pb - cb, pc - kba - ca, \text{ class } 2 \rangle \ (k = 0, 1, \dots, p-1).$$

Over all p values of k , algebra 6.114 has $4p-4$ descendants of order p^7 and p -class 3. The cases $k = -1$ and $k = 3$ are straightforward, but things are more complicated when $k \neq -1, 3$. In these cases we have a parametrized family of algebras

$$\langle a, b, c \mid bac - zbab, pa - ba, pb - cb, pc - kba - ca, \text{ class } 3 \rangle,$$

where (for a given $k \neq -1, 3$) z and z' define isomorphic algebras if the ratios $1 : z$ and $1 : z'$ are in the same orbit of ratios $\alpha : \beta$ under the action

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \rightarrow A \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

where A equals

$$\begin{pmatrix} k-1 & 1 \\ -1 & 0 \end{pmatrix} \text{ or } \begin{pmatrix} k^2-2k & k-1 \\ 1-k & -1 \end{pmatrix} \text{ or } \begin{pmatrix} (1+\gamma k)(\gamma k-2\gamma+1) & \gamma(\gamma k+2-\gamma) \\ -\gamma(\gamma k+2-\gamma) & -(-1+\gamma)(\gamma+1) \end{pmatrix}$$

with $\gamma \neq -1$ and γ not a root of $\gamma^2 + (k-1)\gamma + 1 = 0$. (Note that the ratio $1 : 0$ is in the same orbit as the ratio $0 : 1$.)

A MAGMA program to compute a set of representative pairs (k, z) is given in Notes6.114.m.